# THE HYDRODYNAMICS OF MIXED CRYSTALLIZERS 

L. Musil<br>Chemopetrol, Institute of Inorganic Chemistry, 40060 Ústi nad Labem

Received October 22nd, 1974

The effect was studied of the impeller diameter and its height over the bottom on stirring of model suspensions. The experiments were carried out in a model crystallizer with conical bottom and two types of axial impellers. The experiments covered two regions characterized by the following empirical relations: $\mathrm{Re}_{\mathrm{k}}=$ const., and $\mathrm{Re}_{\mathrm{k}}=c_{1}\left(V_{\mathrm{m}} / V_{\mathrm{c}}\right)+c_{2}$. The relations agree with the hypothesis regarding the correlation between suspensibility of the solids and local intensity turbulence. The experimental results were evaluated from two practical standpoints: the aptitude to mechanical damage to the crystals and the energy requirements.

Crystallization of products plays an important role in many operations of chemical technology. The operation is usually carried out in a mixed crystallizer. Unfortunately, industrial crystallizers often do not meet present requirements as far as the capacity, economy of production and quality of the product is concerned and bottleneck the operation as a whole.
The work reported on in this paper deals with the problem of suitable mixing of crystalline suspensions aimed at proper uniformity of dispersion of crystal particles in mother liquid necessary for regular and sufficiently intensive growth of the crystals. However, care must be taken to prevent in the first place excessively intensive mixing causing mechanical damage to the crystals (this concerns mainly the largest crystals) which could adversely affect the granulometric composition of the product and eventually also the reliability of the equipment. Secondly, the costs on the mechanical energy required for mixing must be kept down.

Dispersion of solid particles in a liquid by mechanical mixing depends to a large extent on physical properties of the phases, geometrical characteristics of the crystallizer and the frequency of revolution of the impeller. This study is focused on the effect of two geometrical variables of the system: the impeller diameter, $d$, and the height of the impeller over the bottom, $h$.

The impellers commonly recommended for mixing of suspensions are those of axial type. Such impellers drive the liquid vertically and almost without exception toward the bottom of the vessel. It is thought that this accomplishes dispersion of solid particles with a minimum energy consumption and it is expected that also with a minimum damage done to the particles.

In view of practical applications it is important to know the so called point of dispersion, i.e. the state when all particles present were just suspended. A common characteristics of this point is the critical frequency of revolution of the impeller, $n_{\mathrm{k}}$, appearing in numerous correlations in
the literature. Two examples of such correlations are given below. Both relations were reduced only to the variables examined in this work:

$$
\begin{align*}
& n_{\mathrm{k}}=k^{\prime} D^{0.82} / d^{1.67}, \quad\left(\text { Zwietering }{ }^{1}\right)  \tag{l}\\
& n_{\mathrm{k}}=k^{\prime \prime} D h^{0.19} / d^{2.09}, \quad\left(\text { Hobler }^{2}\right) \tag{2}
\end{align*}
$$

$D$ designates the diameter of the mixed vessel and $k^{\prime}$, or $k^{\prime \prime}$ the constants incorporating other process parameters. Eqs ( $I$ ) and (2) express in a relatively good agreement the dependence of the critical frequency of the impeller, $n_{k}$, on the impeller diameter, $d$. Both authors worked with propeller stirrers and standardized mixing equipment including flat bottom of the vessel. The approach to the problem in both cases was purely empirical.

## THEORETICAL

The presented model is based on the model concept of the relation between dispersion of solid particles and the turbulence of liquid induced by rotation of the impeller. It will be assumed that in order to reach a full dispersion of all particles present a certain minimum intensity of turbulence, $u_{\mathrm{k}}^{\prime}$, ${ }^{*}$ is required.

First, simplified concept of homogeneous distribution of the turbulent kinetic energy of liquid over the whole volume of the mixed batch will be used. For this purpose we shall use the earlier derived expression ${ }^{3}$ for the intensity of turbulence ${ }^{* *}$ $u^{\prime}$ :

$$
\begin{equation*}
u^{\prime}=K n d^{2} /\left(D^{2} H\right)^{1 / 3}, \quad\left(\operatorname{Re} \leqq 10^{4}\right) \tag{3}
\end{equation*}
$$

where $H$ denotes the clear liquid height. $K$ is a constant.

At the point of dispersion we can write $u^{\prime}=u_{\mathrm{k}}^{\prime}$ and $n=n_{\mathrm{k}}$ and Eq. (3) may be arranged to

$$
\begin{equation*}
\mathrm{Re}_{\mathrm{k}}=n_{\mathrm{k}} d^{2} / v=c \tag{4}
\end{equation*}
$$

where $\mathrm{Re}_{\mathrm{k}}$ represents the appropriate critical value of the Reynolds number, $c$ is a constant incorporating other process parameters. ${ }^{* * *}$

The physical model will be further extended by incorporating the concept of gradual decay of intensity of turbulence in a volume element of liquid as the latter moves away from the impeller. It has been known that mechanical energy supplied by rotat-

* The intensity of turbulence $u^{\prime}$ is defined by $u^{\prime}=\left(u^{2}\right)^{1 / 2}$, where $u$ designates the instantaneous fluctuation component of the turbulent velocity of liquid.
** Eq. (3) was derived from the familiar formula for the rate of dissipation of mechanical energy in a unit volume $\varepsilon \sim \varrho u^{3} / l$. In the given case the characteristic length $/$ is the diameter of the impeller. The effect of the suspended particles in this case is neglected.
*** Eq. (4) is valid for a specific value of the kinematic viscosity.
ing impeller to the mixed batch is consumed giving rise to vortex motion. The original large size vortices disintegrate gradually into smaller vortices of higher frequency. The kinetic energy of turbulence is ultimately transformed by viscous dissipation through the smallest vortices into heat. The process of decay of turbulence is accompanied by decreasing intensity of turbulence. The rate of mechanical energy dissipation in a unit volume may be expressed as ${ }^{4}$

$$
\begin{equation*}
\varepsilon=-3 / 2 \varrho \bar{U} d\left(u^{\prime 2}\right) / \mathrm{d} z, \tag{5}
\end{equation*}
$$

where $\bar{U}$ represents mean velocity of the liquid element and $z$ its instantaneous distance from the source of turbulence - the impeller - in the direction of flow.

It is known that with decreasing length the vortices lose their original directional orientation and it may be justly assumed that vortices equalling in size the diameter of the suspended particles satisfy the condition of isotropic turbulence.

Further it will be assumed that the present solid particles become suspended by the action of the turbulent flow at the expenses of mechanical energy dissipation per unit time in accord with the expression ${ }^{5}$

$$
\begin{equation*}
\varepsilon \sim \varrho u^{\prime 3} / a \tag{6}
\end{equation*}
$$

where $a$ represents a characteristic dimension of the flow. In this case a is the mean diameter of the suspended solid particles.
Comparing both expressions for the rate of energy dissipation (5) and (6) and after some arrangement we obtain

$$
\begin{equation*}
-\left(\bar{U} / u^{\prime 3}\right) \mathrm{d}\left(u^{\prime 2}\right) / \mathrm{d} z=k_{1} / a, \tag{7}
\end{equation*}
$$

where $k_{1}$ is a constant.
Integration of Eq. (7) yields an expression for the intensity of turbulence, $u^{\prime}$, as a function of the distance of the examined volume element of liquid from the source of turbulence (it is assumed that the mean velocity of the liquid element, $\bar{U}$, is roughly independent of the distance $z$ ).

$$
\begin{equation*}
\bar{U} / u^{\prime}=1 / 2 k_{1}(z / a)+k_{2}, \tag{8}
\end{equation*}
$$

where $k_{2}$ is an integration constant.
The system at the point of dispersion will entrain mainly those particles the distance of which from the impeller equals the height of the impeller over the bottom, $z=h$. At this point the intensity of turbulence must reach the critical value, $u_{\mathrm{k}}^{\prime}$, given by the frequency of revolution of the impeller, $n_{k}$. The latter will be assumed to be related to the mean velocity of the liquid ejected from the rotor region, $\bar{U}$, by the
relation

$$
\begin{equation*}
\bar{U} \sim n d \tag{9}
\end{equation*}
$$

Eq. (9) follows from a simplified model of the stream ejected by the blades of the impeller as a stream tube and relates to the mean velocity over its area of cross section.

Substituting from Eq. (9) into (8) and considering the point of dispersion one obtains

$$
\begin{equation*}
n_{\mathrm{k}} d / u_{\mathrm{k}}^{\prime}=\left(k_{1}^{\prime} h / a\right)+k_{2}^{\prime} . \tag{10}
\end{equation*}
$$

Eq. (10) has been arranged to the final form

$$
\begin{equation*}
\operatorname{Re}_{\mathrm{k}}=\operatorname{Re}_{\mathrm{a}}(d / a)\left[\left(k_{1}^{\prime}(h / a)+k_{2}^{\prime}\right)\right] \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
\operatorname{Re}_{\mathrm{u}}=u^{\prime} a / v \tag{12}
\end{equation*}
$$

Eqs (10) and (11) indicate mainly the linear relationship between the critical frequency of revolution of the impeller, $n_{k}$, and the height of the impeller over the bottom, $h$.

## EXPERIMENTAL

Experimental set-up. The model crystallizer used for experiments is an open cylindrical vessel with conical bottom and four vertical baffles. At the apex of the bottom there is a discharge opening. The inner diameter of the vessel is $D=0.6 \mathrm{~m}$, the apex angle of the conical botton $x=120^{\circ}$. In the vertical axis of the vessel there is a shaft with interchangeable impeller. Two types of axial impellers with flat inclined blades were used: a three-blade impeller with the inclination angle of the blades $\beta=24^{\circ}$ and a six-blade impeller, $\beta=45^{\circ}$. The diameter of the two types of impellers equaled for the three-blade impeller $d=0.20 ; 0.25 ; 0.315 \mathrm{~m}$ and for the six-blade impeller $d=0.15 ; 0.195 ; 0.245 \mathrm{~m}$. The r.p.m. of the impeller could be varied between zero and $700 \mathrm{~min}^{-1}$. The vertical position of the impeller could be arbitrarily adjusted within the cylindrical vessel.

A crystalline suspension was simulated by a mixture of water and screened fractions of glass balotini. The diameter of the beads was $a=1.0-1.2 \mathrm{~mm}$ with the mean diameter of the fraction 1.1 mm and $a=0.6-0.8 \mathrm{~mm}$ with the mean diameter of the fraction 0.7 mm . The density of the particles was $\varrho=2.7 .10^{3} \mathrm{~kg} / \mathrm{m}^{3}$. The volume concentration of the particles in mixture was 0.04 . The height of the mixture was $H=D=0.6 \mathrm{~m}$ (about $0.180 \mathrm{~m}^{3}$ ). The temperature of the mixed batch was held at $t=25^{\circ} \mathrm{C}$.

Critical frequency for stirving a suspension. This quantity is currently defined as a minimum frequency of revolution of the impeller required for suspending all present solid particles in the mixed liquid.

Owing to the conical bottom of the vessel, which is typical for crystallizers, the above definition cannot be used because certain part of the solids remains within the apex of the cone even under the optimum regime of stirring (low position of the impeller, maximum r.p.m.).

In this work we have adopted a difierent definition of the point of dispersion. It is based on the observation of concentration of the suspension in a certain point above the level reached by the particles in the quiescent liquid ${ }^{6}$. With increasing r.p.m. ever larger portions of the solids resting originally on the bottom are being suspended causing the concentration of the particles in the given point proportionately to increase. At a certain frequency a sudden change occurs: The concentration begins to decrease (small diameter impellers) or keeps on increasing but at a significantly slower rate (large diameter impellers). The break on the concentration curve indicates exhaustion of suspendable particles on the bottom of the vessel, i.e. the state which shall be termed the point of dispersion and the corresponding frequency; the critical frequency, $n_{\mathrm{k}}$.

As we have mentioned certain part of the solids remains in the apex of the conical bottom even under conditions above the point of dispersion. From visual observations we have concluded that the diameter of the circular top surface of the layer of unentrained particles roughly agrees with the diameter of the impeller used, see Fig. 1.

Determination of the point of dispersion. The determination of the point of dispersion was carried out by observing concentration of the model suspension in a small volume near the wall of the vessel. An optical method based on absorption of light was used. The measuring device consisted of a light source, photodiode, measuring circuit, recorder and an integrator. The light source was located outside the vessel throwing the optical beam through the wall. The source was powered by a stabilized source of voltage. The photodiode was mounted in a watertight stainless steel chamber with a glass window and immersed into the mixed suspension. Position


Fig. 1
Residual Solids Trapped in Apex of Conical Bottom at the Point of Dispersion with Impeliers of Various Diameters


Fig. 2
Course of Local Volume Concentration of Suspension as a Function of Re

Three-blade 0.2 m impeller located at the level of the lower edge of the vessel. Sclid particles 1.1 mm mean diameter at the mean concentration of suspension 0.04. Measured at four $h_{\mathrm{m}}(\mathrm{m})$ : $10.05 ; 20.15:, 30.25: 4$ 0.35 .
of the photodiode was standardized at 25 mm from the wall of the vessel 50 mm above the lower edge of the cylindrical part of the crystallizer half way between two baffles. The signal of the photodiode was recorded on a chart recorder and, owing to the large fluctuations of the measured concentration in the examined volume, averaged by means of an integrator.

The point of dispersion and associated critical frequency of the impeller determined as mentioned above depend on the location of the impeller in the investigated region. The key role is played by the vertical height of the probe. It may be assumed in the extreme case of observing the concentration of the suspension in the immediate vicinity of the liquid level that the indicated point of dispersion would correspond to complete homogenation of the suspension for monodisperse particles.

The effect of vertical position of the probe was tested on four horizontal levels and the same vertical straight line $h_{\mathrm{m}}=50 ; 150 ; 250 ; 350 \mathrm{~mm}$ above the lower edge of the cylindrical part of the vessel. The tests were carried out under otherwise identical conditions: three-blade impeller of $d=0.2 ; 0.25 ; 0.315 \mathrm{~m}$ located gradually at three different locations above the lower edge of the cylindrical part of the vessel: $h_{2}=0 ; 0.1 ; 0.2 \mathrm{~m}$. Glass balotini of the mean particle diameter $a=1.1 \mathrm{~mm}$ was used. The volume concentration of the suspension was $s=0.04$.

Typical course of local concentration of the suspension as a function of the Reynolds number is shown for two cases: $d=0.2 \mathrm{~m} ; h_{2}=0$ in Fig. 2 and $d=0.315 ; h_{2}=0.1 \mathrm{~m}$ in Fig. 3. Both figures show by arrows the locations of anticipated points of dispersion. These critical states are less obvious in Fig. 3 which is probably associated with the relatively larger number of immobile particles trapped in the apex of the cone even after the full dispersion had been reached with the


Fig. 3
Course of Local Volume Concentration of Suspension as a Function of Re

Three-blade 0.3 m impeller located 0.1 m above the lower edge of the vessel. Solid particles 1.1 mm mean diameter at the mean concentration of suspension 0.04 . Measured at four $h_{\mathrm{m}} / m,: 10.05 ; 20.15 ; 30.25 ; 40.35$.


Fig. 4
Effect of Relative Height of the Probe, $h_{\mathrm{m}} / D$, on the Derived Critical Value of the Reynolds Number

Three-blade impeller. 1.1 mm solid particles at the mean concentration of suspension $0.04 .11 d=0.2 m, h_{2}=0 ; 120.2 ; 0.1 ; 21$, $0.25,0 ; 220.25,0.1 ; 230.25,0.2 ; 310.315$, $0 ; 320.315,0.1$.
aid of large diameter impellers (Fig. 1). With the frequency of the impeller growing above the critical value the trapped particles are gradually entrained.

The measured critical values of the Reynolds number are plotted in Fig. 4.
On the basis of the just described measurements the following inferences can be drawn: For the relative height $n_{\mathrm{m}} / D$ ranging between 0.083 and 0.25 the recorded value of the critical frequency of the impeller remained virtually constant. In the region of $h_{\mathrm{m}} / D=0.25-0.417$ the critical value significantly grew. No critical value was detected by our experimental technique for the relative height $h_{\mathrm{m}} / D$ equal 0.583 .

With regard to the determination of the critical frequency corresponding to the earlier described point of dispersion the relative height of the probe, $h_{\mathrm{m}} / D$, should not exceed 0.25 as suggest the experimental measurement. In this work we used a standard height $h_{\mathrm{m}} / D=0.083$.

## RESULTS

The measured critical frequency of revolution of the impeller was correlated according to Eq. (13) which is linear with respect to the height of the impeller $h=$ $=h_{1}+h_{2}$ just like the theoretically derived expression (11). These two equations though differ in the exponent over the diameter of the impeller, $d$.

$$
\begin{equation*}
\operatorname{Re}_{\mathrm{k}}=c_{1}\left[d^{2}\left(h_{1}+h_{2}\right) / D^{3}\right]+c_{2}, \tag{13}
\end{equation*}
$$

$c_{1}$ and $c_{2}$ are constants incorporating the effect of the remaining variables.
The height of the impeller, $h_{1}$, corresponding to the conical part of the vessel is given by the geometry of the equipment:


Fig. 5
Critical Reynolds Number of the Point of Dispersion, $\mathrm{Re}_{k}$, as a Function of the Relative Volume of Cylinder ( $V_{\mathrm{m}} / V_{\mathrm{c}}=d^{2} h / D^{3}$ ),

Three-blade impellers. $\circ d=0.2 \mathrm{~m}$, $a=1.1 \mathrm{~mm} ; 0.2,0.7 ; 0.25,1.1 ; 0.25$, $0.7 ; 0.315,1.1 ; \ominus 0.315,0.7 . \longrightarrow a=$ $=1.1 \mathrm{~mm},-\cdots--a=0.7 \mathrm{~mm}$.


Fig. 6
Critical Reynolds Number of the Point of Dispersion, $\mathrm{Re}_{\mathrm{k}}$, as a Function of the Relative Volume of Cylinder ( $V_{\mathrm{m}} / V_{\mathrm{c}}=d^{2} h / D^{3}-$ $-d^{2} / 6 D^{2}$ ).

Six-blade impellers, $O d=0.15 \mathrm{~m}, a$ a 1.1 mm ; 0.15, 0.7; 0.195, 1.1; 0.195, 0.7; $\ominus 0.245,1.1 ; \ominus 0.245,0.7$. ——a=1.1 $\mathrm{mm}----a=0.7 \mathrm{~mm}$.

$$
\begin{equation*}
h_{1}=(D-d) / 2 \operatorname{tg} \alpha \tag{14}
\end{equation*}
$$

The fraction on the right hand side of Eq. (13) represents the relative volume of a cylinder with the diameter equal the diameter of the impeller and the height equal the height of the impeller $h=h_{1}+h_{2}$ as a fraction of the volume of the cylindrical part of the vessel: $V_{\mathrm{m}} / V_{\mathrm{c}}$.

Figs 5 and 6 show graphically the correlations of the experimental data for the three- and the six-blade impeller according to the resultant equation as

$$
\begin{equation*}
\operatorname{Re}_{\mathrm{k}}=c_{1}\left(V_{\mathrm{m}} / V_{\mathrm{c}}\right)+c_{2} . \tag{15}
\end{equation*}
$$

For the six-blade impeller we had to use a modified relation for the height of the impeller $h=h_{1}+h_{2}-0 \cdot 1$, ( $h$ in meters), which suited better.

The dependences shown in Figs 5 and 6 have a linear course with breaks. The left hand side part corresponds roughly to

$$
\begin{equation*}
\mathrm{Re}_{\mathrm{k}}=\mathrm{const} \tag{16}
\end{equation*}
$$

Table I summarizes the coordinates of the point of break of these straight lines.
The coefficients of the slanting part of the dependences in Fig. 5 and $6, c_{1}$ and $c_{2}$ determined by the least square method, are also shown in Table I.

From a statistical processing of the data it follows that Eq. (15) with the coefficients given in Table I suits well for both types of impellers for the slanting part of the experimental dependence.

## DISCUSSION

The fitting equation (13) roughly confirms the model concept of the effect of the decay of turbulence on the process of suspending the solid particles as it is evidenced by the linear course of the dependence of the critical frequency of the impeller on its height.

In contrast to the model equation (11), the fitting equation (13) has a different exponent over the diameter of the impeller. For the correlation we used the second power although even this value does not perfectly suit the experimental data.*

The horizontal parts of the curves correspond to the condition (16) and may be explained within the framework of this model as a region immediately behind the impeller where the turbulent energy is not being fully dissipated as yet. In these

* The values of the coefficients $c_{1}$ and $c_{2}$ given in Table I are in three out of four cases more or less a function of the diameter of the impeller, $d$. This is associated with the fact that for the whole groups of data the calculated mean values of the above coefficients appear in some cases outside the interval of the partial values of the coefficients. The determingtion of the dependence of the critical frequency of the impeller on impeller*s diameter to a higher precision will be subject of next study.
Table I
The Characteristics of the Experimental Dependence Given in Eqs (15) and (16)

| $\begin{gathered} d \\ \mathrm{~m} \end{gathered}$ | $\begin{gathered} a \\ \mathrm{~mm} \end{gathered}$ | Coordinate of the point of break |  | Coefficients of the slanting section |  |  |  |  |  |  | $\delta_{c_{2}}^{\prime}$ | $\omega$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{Re}_{\mathrm{k}}$ | $V_{\mathrm{m}} / V_{\mathrm{c}}$ | $c_{1}$ | $c_{2}$ | $\delta_{c_{1}}$ | $\delta_{c_{2}}$ | $\bar{c}_{1}$ | $\bar{c}_{2}$ | $\delta_{c_{1}}^{\prime}$ |  |  |
| Three-blade impeller |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.20 | $1 \cdot 1$ | $1.68 .10^{5}$ | 0.040 | 35.2 | 0.278 | 1.0 | 0.045 |  |  |  |  |  |
| 0.25 |  | $1.99 .10^{5}$ | 0.051 | 29.9 | 0.441 | 0.9 | 0.065 | $26 \cdot 1$ | 0.735 | $1 \cdot 3$ | 0.079 | 0.984 |
| 0.315 |  | - | - | 23.0 | 0.981 | $1 \cdot 1$ | 0.066 |  |  |  |  |  |
| $0 \cdot 20$ | 0.7 | $1 \cdot 57.10^{5}$ | 0.035 | 28.5 | 0.547 | 0.9 | 0.043 |  |  |  |  |  |
| 0.25 |  | $1.76 .10^{5}$ | 0.043 | 26.2 | 0.622 | 0.7 | 0.047 | $25 \cdot 8$ | 0.675 | 0.5 | 0.030 | 0.997 |
| 0.315 |  | - | - | $25 \cdot 1$ | 0.753 | 0.6 | 0.042 |  |  |  |  |  |
| Six-blade impeller |  |  |  |  |  |  |  |  |  |  |  |  |
| $0 \cdot 15$ | 1.1 |  | 0.013 | $41 \cdot 1$ | 0.553 | 3.6 | $0.058$ |  |  |  |  |  |
| $0 \cdot 20$ |  | $1 \cdot 20.10^{5}$ | 0.017 | $35 \cdot 7$ | $0.609$ | 0.6 | $0.015$ | $34 \cdot 2$ | 0.651 | $1 \cdot 0$ | 0.022 | 0.996 |
| 0.245 |  | - | - | 34.9 | 0.632 | - | - |  |  |  |  |  |
| 0.15 | 0.7 | $1.05 \cdot 10^{5}$ | 0.013 | 31.0 | 0.668 | 0.9 | 0.018 |  |  |  |  |  |
| 0.20 |  |  | - | $33 \cdot 1$ | 0.550 | 0.9 | $0.027$ | 28.6 | 0.674 | 1.5 | 0.043 | 0.984 |
| 0.245 |  | $0.99 .10^{5}$ | 0.016 | 33.6 | 0.441 | 0.9 | 0.033 |  |  |  |  |  |

regions where the height of the impeller plays no role in affecting the critical value of the Reynolds number one can use the simplified model in Eq. (4).

## Gentleness of Dispersion of Crystalline Suspensions

The aptitude to breaking large solid particles (crystals) in the mixed volume will no doubt depend on the magnitude of the impulse, $\Delta I_{\mathrm{k}}$, impared to the particle by the rotating blade of the impeller

$$
\begin{equation*}
\Delta \mathbf{I}_{\mathrm{k}}=\boldsymbol{P} \Delta t=m \Delta \mathbf{v} \tag{17}
\end{equation*}
$$

where $\boldsymbol{P}$ designates the vector force of impact of the blade, $\Delta t$ is the time of duration of the impact on the particle, $m$ is the mean mass of the solid particle and $\Delta \boldsymbol{v}$ is the increment of the velocity vector of the particle.

On adopting the assumption that the solid particle prior to the impact was at rest and that the velocity of the impact is the component $v_{1 \mathrm{k}}$ of the velocity of revolution of the blade perpendicular to the particle's surface we can write

$$
\begin{equation*}
v_{1 \mathrm{k}}=\left|\boldsymbol{v}_{\mathrm{c}}\right| \sin \beta \tag{18}
\end{equation*}
$$

where $\beta$ is the angle of inclination of the blade with respect to the horizontal plane and $v_{c}$ is the peripheral velocity of the blade at the point with the radical coordinate $r$ given by

$$
\begin{equation*}
\left|\mathbf{v}_{\mathrm{e}}\right|=2 \pi r n \tag{19}
\end{equation*}
$$

The absolute value of increment of the particle's velocity during the impact is then

$$
\begin{equation*}
\Delta v=2 v_{\mathrm{e}} \sin \beta \tag{20}
\end{equation*}
$$

Substituting from Eqs (18)-(20) into (17) and considering that we are interested in the critical values of the momentum $\Delta I_{\mathrm{k}}$ and the frequence of the impeller $n_{\mathrm{k}}$, one obtains for the absolute magnitude

$$
\begin{equation*}
\Delta I_{\mathbf{k}}=2 \pi m n_{\mathbf{k}} d \sin \beta \tag{21}
\end{equation*}
$$

The last equation is an expression for the maximum impulse imparted to the particle after a collision with the outer edge of the rotating blade of the impeller.

As we are concerned with the hydrodynamic conditions of mixing of the model suspensions from the viewpoint of a minimum impulse imparted to the solid particles it suffices to work with the relative values. Eq. (21) then transforms into

$$
\begin{equation*}
\Delta I_{\mathrm{krel}}=\frac{\Delta I_{\mathrm{k}}}{\Delta I_{\mathrm{ks}}}=\frac{d_{\mathrm{s}}}{d} \frac{\operatorname{Re}_{\mathrm{k}}}{\operatorname{Re}_{\mathrm{ks}}} \tag{22}
\end{equation*}
$$

and the subscript $s$ refers to standard conditions arbitrarily chosen.

The functions in Eq. (22) are presented with the aid of the empirical relation (15) for $\mathrm{Re}_{\mathrm{k}}$ for the three-bladed impellers in Fig. 7 and for the six-bladed impellers in Fig. 8.
The plotted dependences $\Delta I_{\text {kre }}$ versus $V_{\mathrm{m}} / V_{\mathrm{c}}$ display breaks on their course. As the coordinates of the points of the break we used the mean values given in Figs 5 and 6. The broken lines indicate corresponding height of the impeller over the lower edge of the cylindrical part of the crystallizer, $h_{2} / D$.
Figs 7 and 8 provide information about the effect of the hydrodynamic conditions on the magnitude of the impulse imparted by the outer edges of the impeller's blades to the solid particle: The effect of impeller diameter, $d: a$ ) The diameter of the impeller has a considerable effect on the magnitude of the momentum in region described by Eq. (16). b) The effect of the impeller's diameter is only weak in the experimental region described by Eq. (15). The effect of the type of impeller: Both tested types of impellers were compared under the standard conditions and it was found that $\Delta I_{\mathrm{ks} 6} / \Delta I_{\mathrm{ks} 3}=1.442$.


Fig. 7
Effect of Hydrodynamic Conditions on Relative Value of the Impulse, $\Delta /_{\text {krel }}$, Imparted by the Impeller to the Solids at Critical Frequencies for Three-Blade Im* pellers


Fic. 8
Effect of Hydrodynamic Conditions on Relative Value of the Impulse, $\Delta I_{k r e l}$, Imparted by the Impeller to the Solids at Critical Frequencies for Six-Blade Impellers

The right-hand side abscissas refer to the relative value of the impulse, $\Delta l_{\text {kre13 }}$, derived for the three-blade impellers.

The relative values of the impulse related to the standard conditions used for the three-blade impeller are plotted right in Fig. 8. A comparison of the corresponding dependences in Figs 7 and 8 indicates the advantage of the three-blade impellers in view of their lower values of the impulse imparted to the particles at the critical frequencies of revolution.

## The Economy of Dispersing Crystalline Suspensions

The question is that of the effect of hydrodynamic conditions of stirring of crystalline suspensions on the appropriate energy input. The specific value of this quantity corresponding to a unit volume of suspension is given by

$$
\begin{equation*}
\varepsilon=\text { Eugn } n^{3} d^{5} / V_{c} \tag{23}
\end{equation*}
$$

where Eu is the Euler power input criterion. For a given type of the impeller* we thus have

$$
\begin{equation*}
\varepsilon_{\mathrm{kreI}}=\varepsilon_{\mathrm{k} /} / \varepsilon_{\mathrm{ks}}=d_{\mathrm{s}} \mathrm{Re}_{\mathrm{k}}^{3} / d \mathrm{Re}_{\mathrm{ks}}^{3} \tag{24}
\end{equation*}
$$



* In this context we neglect the dependence of the power-input criterion Eu on the height of the impeller. The error commited does not exceed $10 \%$ as shown by preliminary calculations.

The dependence according to Eq. (24) used jointly with the empirical expression (15) for $\mathrm{Re}_{\mathrm{k}}$ is presented for the three-blade impellers in Fig. 9 and for the six-blade impellers in Fig. 10. The plotted dependences of $\varepsilon_{\mathrm{krel}}$ versus $V_{\mathrm{m}} / V_{\mathrm{c}}$ display again the broken course in accord with the broken course of the experimentally found dependence of $\operatorname{Re}_{k}$ versus $V_{m} / V_{c}$.

Figs 9 and 10 illustrate these findings regarding the effect of the hydrodynamic conditions on the calculated energy required for dispersing the model suspension:

The effect of the impeller's diameter, $d$; the effect of the impeller's diameter on the theoretical energy input is almost always significant. However, in region described by Eq. (16) is positive and in region described by Eq. (15) negative.
The effect of the type of the impeller: The ratio of the standard values for both types of impellers tested in this work is $\varepsilon_{\text {ks } 6} / \varepsilon_{\text {ks } 3}=2 \cdot 1$ Fig. 10 plots on the right-hand side the relative values of the critical energy inputs related to the standard conditions used for the three-blade impellers. A comparison of Figs 9 and 10 reveals the advantage of the three-bladed impellers with regard to their lower specific energy input required to disperse a given suspension.

The author wishes to thank his coworkers, employees of the department of processes and apparatuses of the Institute of Inorganic Chemistry, Ústi nad Labem: Mr S. Procházka for his assistance in constructing the optical device; the assistance of Mr J. Vlk and Mr L. Provaznik in performing the experiments is also acknowledged.

## LIST OF SYMBOLS

| $a$ | diameter of suspended particle |
| :--- | :--- |
| $\bar{c}$ | mean value of coefficient $c$ |
| $d$ | diameter of impeller |
| $D$ | diameter of mixing vessel |
| $h$ | height of impeller over bottom |
| $h_{2}$ | height of impeller over lower edge of cylindrical part of mixing vessel |
| $h_{\mathrm{m}}$ | height of probe over lower edge of cylindrical part of mixing vessel |
| $H$ | clear liquid height in vessel |
| $m$ | mass of suspended particle |
| $n$ | frequency of revolution of impeller |
| $n_{\mathrm{k}}$ | critical frequency of revolution of impeller corresponding to the point of dispersion |
| $s$ | volume concentration of suspension |
| $u^{\prime}$ | intensity of turbulence |
| $U$ | velocity of liquid ejected by blades of impeller |
| $t$ | velocity of suspended solid particle |
| $V=\pi 4 D_{\mathrm{c}}^{3}$ | volume of batch contained in the cylindrical part of mixing vessel |
| $V=\pi / 4 d^{2} h$ | volume of cylinder |
| $z$ | vertical distance from impeller |
| $\alpha$ | apex angle of conical bottom of mixing vessel |
| $\beta$ | angle of inclination of impeller blades |
| $\delta_{\mathrm{c}}$ | standard deviation of $c$ |


| $\delta_{\mathrm{c}}^{\prime}$ | standard deviation of a group of data on $c$ |
| :--- | :--- |
| $\varepsilon$ | rate of energy input for mixing per unit volume of batch |
| $\varrho$ | density of liquid phase |
| $v$ | kinematic viscosity of suspension |
| $\omega$ | correlation coefficient |
| $\operatorname{Re}=n d^{2} / v ;$ | $\operatorname{Re}_{\mathrm{k}}=n_{\mathrm{k}} d^{2} / v ; \quad \mathrm{Re}_{a}=u^{2} a / v$ |

## REFERENCES

1. Zwietering T. N.: Chem. Eng. Sci. 8, 244 (1958).
2. Hobler T., Zablocki J.: Chem. Stosowana 3B, 265 (1965).
3. Schwartzberg H. G., Treybal R. B.: Ind. Eng. Chem. Fundam. 7, 1 (1968).
4. Taylor G. I.: Proc. Roy. Soc., Ser. A /51, 421 (1935).
5. Kolář V.: This Journal 32, 526 (1967).
6. Rushton J. H., Fridley D. L.: Thesis. Purdue University, Lafayette.
7. Kvasnička J.: Thesis. Institute of Chemical Technology, Prague 1967.

Translated by V. Stanék.

